Quantitative sum rule analysis of low-temperature spectral functions

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In vacuum and at low temperatures, the chiral symmetry of the QCD Lagrangian is spontaneously broken by a non-vanishing quark condensate. For isovector hadronic excitations, this manifests itself through a large splitting of the masses of the chiral partner mesons $\rho(770)$ and $a_1(1260)$. Chiral symmetry is believed to be gradually restored with rising temperatures and baryon densities, as the quark condensate "melts" away. The melting is expected to be accompanied by a disappearance of the mass splitting of chiral-partner hadrons, i.e., the ρ and a_1 spectral functions should degenerate when chiral symmetry is restored. A model-independent prediction for the low-temperature behavior of the spectral functions is chiral mixing [1], where the medium constitutes a lukewarm pion gas: To leading order in temperature, a mutual linear mixing of the vector and axial-vector spectral functions occurs. Further consequences of this mechanism can be explored using spectral representations in finite-temperature sum rules.

In the present work [2] we have employed chiral mixing to quantitatively analyze finite-temperature QCD and Weinberg-type sum rules. In particular, we have utilized: (i) updated vacuum spectral functions which quantitatively agree with sum rules and include both excited resonances and degenerate continua [2], (ii) finite pion-mass corrections, and (iii) a strict implementation of leading-order temperature effects. Chiral mixing induces a mutual flattening of the oscillatory pattern of "peaks" and "valleys" in the vacuum spectral functions, with the peaks in one channel filling in the valleys in the opposite channel. The in-medium spectral distributions thus tend to approach one another, as expected with increasing temperature. It further has been found that, while the Weinberg-type sum rules are trivially satisfied by leading-order chiral mixing, the QCD sum rules begin to break down at temperatures near 140 MeV and above, cf. Table I and Fig. 1 (some sensitivity to the definition of the Borel window at finite temperature is observed). Interestingly, the breakdown sets in at much higher temperatures when finite pion mass corrections are neglected. The latter are thus identified as a key factor in signaling the onset of new physics. It is encouraging to find this onset at temperatures where one typically expects corrections from higher resonances to become important.

Table I. Average deviation, $d_{V,A}$, of the QCD sum rule over the Borel window for axial-/vector channels at select temperatures (the second line quotes the chiral-mixing parameter, with ε=1/2 corresponding to chiral restoration). Values in parentheses are based on a frozen Borel window equal to the vacuum one.

T (MeV)	0	100	110	120	130	140	150	160	170	180
ϵ	0	0.06	0.08	0.10	0.13	0.16	0.20	0.23	0.28	0.32
$d_V(\%)$	0.24	0.32(0.29)	0.38(0.33)	0.48(0.39)	0.64(0.51)	0.85(0.64)	1.11(0.74)	1.43(0.97)	1.82(1.17)	2.29(1.39)
$d_A(\%)$	0.56	0.65(0.57)	0.70(0.58)	0.78(0.61)	0.90(0.67)	1.08(0.76)	1.30(0.88)	1.60(1.01)	1.98(1.17)	2.53(1.34)

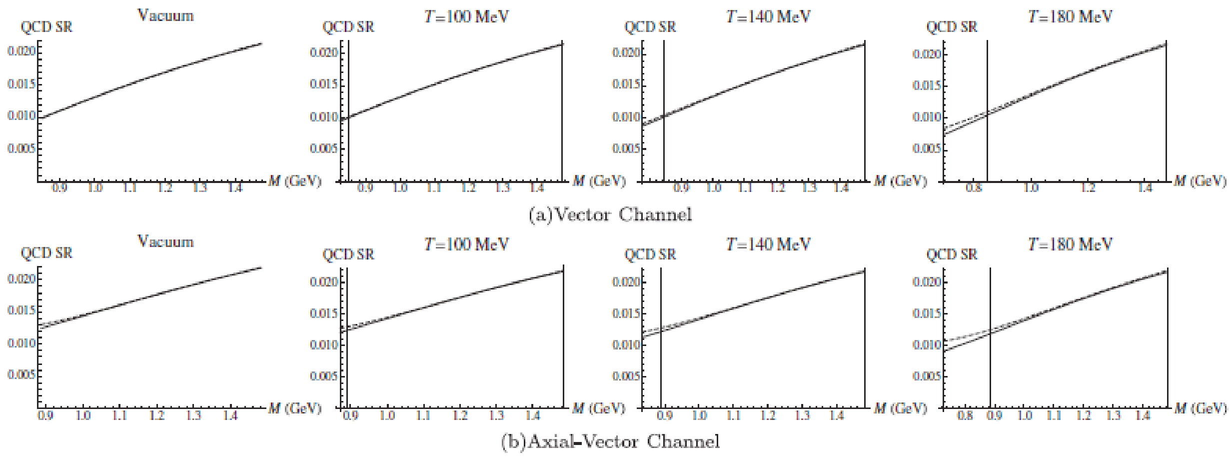

FIG. 1. Comparison of LHS (solid curve) and RHS (dashed curve) of the QCD sum rule for vector (upper panels) and axial-vector (lower panels) at select temperatures. The x-range of each plot indicates the Borel window at that temperature, while the vertical lines designate the Borel window in vacuum.

[1] M. Dey, V.L. Eletsky, and B.L. Ioffe, Phys. Lett. B **252**, 620 (1990).
[2] N.P.M. Holt, P.M. Hohler, and R. Rapp, Phys. Rev. D **87**, 076010 (2013).
[3] P.M. Hohler and R. Rapp, Nucl. Phys. **A892**, 58 (2012).